APRIL/MAY 2019

MPH11 — MATHEMATICAL PHYSICS — I

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Define a linear vector space. And prove that the set of all complex numbers forms a linear vector space over a complex field.

Or

(b) Reduce the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ into a diagonal matrix.

2. (a) Define the Kronecker delta symbol and discuss its properties.

Or

(b) With an example, explain the quotient rule in tensor analysis.

3. (a) Obtain the generating function of $J_n(x)$.

Or

(b) Find the series solution around an ordinary point of the differential equation $\frac{d^2y}{dx^2}+(\lambda-x^2)y=0$, where λ is a constant.

4. (a) Find the Green's function required for the boundary value problem $\frac{d^2y}{dx^2}+k^2y=f(x)$ where $f(x)$ is a known function of x and $y(x)$ satisfies the boundary conditions $y(0)=0$ and $y(L)=0$.

Or

(b) Obtain the eigen function expansion of Green's function.

5. (a) Obtain the moment generating function of binomial distribution.

Or

(b) Fit a Poisson's distribution to the following data and calculate theoretical frequencies :

Birth :	0	1	2	3	4
Frequency :	122	60	15	2	1

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Explain the Gram-Schmidt orthogonalization process to obtain the mutually orthonormal vectors and use it to obtain the same for the following linearly independent set of n- tuples : $\psi_1=(1,0,0,\ldots 0), \psi_2=(1,1,0,\ldots 0), \psi_3=(1,1,1,\ldots 0)$ and $\psi_n=(1,1,1,\ldots 1)$

7. Obtain the tensor form of the operators Gradient, Divergence, Laplacian and Curl.

8. Prove the following recurrence relations for Laguerre polynomials :

(a) $(n+1)L_{n+1}(x)+(2n+1-x)L_n(x)-L_{n-1}(x)$.

(b) $xL_n'(x)+nL_n(x)-nL_{n-1}(x)$.

(c) $L_n^{'(x)}=-\sum_{r=0}^{n-1}L_r(x)$.

9. Explain the Green's function method of solving one dimensional Sturm — Liouville type problems.

10. (a) Derive the normal distribution as the limiting case of binomial distribution.

(b) Obtain the standard form of the normal curve and discuss its properties.