

APRIL/MAY 2018

MPH11 — MATHEMATICAL PHYSICS – I

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL the questions.

1. (a) Explain the concepts of linear independence, basis and dimension in a linear vector space.

Or

- (b) What are unitary matrices? Prove that
- (i) determinant of a unitary matrix can only have values +1 or -1 and
 - (ii) products of unitary matrices are also unitary.

2. (a) Write a note on the coordinate transformation and summation convention in tensor analysis.

Or

- (b) What are invariant tensors? Prove that Kronecker delta is an invariant mixed tensor of rank two.

3. (a) Analyse the singular points of the equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \left(a - \frac{m^2}{1-x^2}\right)y = 0.$$

Or

- (b) State and prove the orthogonal property for Laguerre polynomials.
4. (a) Determine the Green's function associated with $\frac{d^2y}{dx^2} - k^2y = f(x)$ where $f(x)$ is a known function and $y(\pm\infty) = 0$.

Or

- (b) Obtain the eigen function expansion of Green's function.
5. (a) Give an account on the moment generating function of normal distribution.

Or

- (b) Discuss the different discrete distributions.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) What is meant by similarity transformation? Prove that under similarity transformation
- a matrix equation retains its form and
 - the trace of a matrix remains invariant.

- (b) What are orthogonal matrices? Prove that
- the determinant of an orthogonal matrix can only have values +1 or -1.
 - the products of orthogonal matrices are also orthogonal and
 - $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are orthogonal matrices.

7. (a) With an example, explain the contraction of tensors.
- (b) State and explain the Quotient rule in tensor analysis.
8. Obtain the series solution of Legendre's equation.
9. Discuss the Green's function method for solving Sturm — Liouville type equation in one dimension.
10. Write a note on the Binomial distribution and determine its first four moments.