8. Using Laplace transform technique, find the solution of the following initial value problems:

(a) $y''' - 2y'' + 5y' = 0$ with $y(0) = y'(0) = 1$ at $t = 0$ and $y = 1$ at $t = \frac{\pi}{8}$.

(b) $y''' - 3y'' + 3y' - y = t^2 e^t$ with $y(0) = 1$, $y'(0) = 0$ and $y''(0) = -2$.

9. Give an account on the three-dimensional rotation group SO(3).

10. (a) Derive the Doppler's relativistic formula for light waves in vacuum.

(b) Explain the longitudinal and transverse Doppler effects.

(c) How is the Doppler's effect confirmed? Explain.

APRIL/MAY 2019

MPH21 — MATHEMATICAL PHYSICS – II

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z.

Or

(b) Expand $\frac{1}{z^2 - 3z + 2}$ in the region (i) $|z| < 1$ (ii) $|z| > 2$.

2. (a) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge $y = 0$ is given by

$$u(x, 0) = \begin{cases} 20x, & 0 < x < 5 \\ 20(10 - x), & 5 < x < 10 \end{cases}$$

while the two long edges $x = 0$ and $x = 10$ as well as the other short edges are kept at 0°C. Find the steady state temperature at any point (x, y) of the plate.

Or

(b) Find the deflection $u(x, y, t)$ of a rectangular membrane, $0 \le x \le a$, $0 \le y \le b$, given that its entire boundary is fixed, initial velocity is zero (starts from rest) and initial deflection $f(x, y) = kxy(a - x)(b - y)$.

3. (a) Find Fourier sine and cosine transform of (i) $x^{(n-1)}$ (ii) $\frac{1}{\sqrt{x}}$.

Or

(b) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for $x \ge 0$, $t \ge 0$, under the given conditions $u = u_0$ at $x = 0$, $t > 0$ with initial conditions $u(x, 0) = 0$, $x \ge 0$.

4. (a) State and prove the orthogonality theorem.

Or

(b) Explain the steps involved in constructing a character table.

5. (a) Derive the relativistic relation for variation of mass with velocity.

Or

(b) Obtain the Lorentz transformation of space and time in four vector form.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Using the residue theorem

(a) show that $\int_0^{2\pi} \frac{d\theta}{(a + b\cos\theta)^2} = \frac{2\pi a}{(a^2 - b^2)^{\frac{3}{2}}}$ and

(b) evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(1 + x^2)^3}$.

7. If the surface of the sphere of radius 'a' is kept at a fixed distribution of electric potential of the form $u = F(\theta)$, then find the potential inside and outside the spherical surface assuming the space inside and outside the sphere to be free of charges.