9. (a) Obtain the eigen function expansion of Green's function.

(b) Determine the Green's function associated with the boundary value problem $\frac{d^2y}{dx^2} = f(x)$, where $f(x)$ is a known function and $y(0) = 0$ and $y'(1) = 0$, using the method of expansion of Green's function in terms of orthonormal function of operator $\frac{d^2}{dx^2}$.

10. (a) Write a note on the Poisson's distribution and determine its first four moments.

(b) Show that if X_1 and X_2 be two independent random variables with Poisson's distributions with parameter m_1 and m_2 respectively, then the sum $X_1 + X_2$ is a random variable with Poisson's distribution with parameter $m_1 + m_2$.

NOVEMBER/DECEMBER 2019

MPH11 — MATHEMATICAL PHYSICS - I

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) What are linear operators? Define the inverse of an operator and prove that if $\hat{A}$ is a linear operator and is invertible, then $\hat{A}^{-1}$ is also a linear operator.

Or

(b) Explain the Gram-Schmidt orthogonalization process to obtain the mutually orthogonal vectors.

2. (a) With an example, explain the quotient rule in tensor analysis.

Or

(b) Obtain the tensor form of the operator Laplacian.

3. (a) State and prove the orthogonal property of Laguerre polynomials.

Or

(b) Prove that $nP_n = (2n-1)\, xP_{n-1} - (n-1)P_{n-2}$.

4. (a) Prove the symmetry property of Green's function.

Or

(b) Find the Green's function for the boundary value problem $\frac{d^2y}{dx^2} - k^2y = f(x)$, where $f(x)$ is a known function and $y(\pm\infty) = 0$.

5. (a) Obtain the moment generating function of normal distribution.

Or

(b) Give an account on the binomial distribution and determine the first moment of it.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. The orthonormal set of vectors $\varphi_1, \varphi_2, \varphi_3$ form a basis in a linear vector space. An operator $\hat{A}$ such that it transforms φ_1 to φ_2, φ_2 to φ_3 and φ_3 to φ_1.

(a) Find the matrix representing the operator in the given basis.

(b) Is the operator Hermitian or Unitary?

(c) What are the eigen values and eigen vectors of the operator?

7. Define the Levi — Civita symbol ϵ_{ijk} and prove that

(a) $\delta_{ii} = 3$

(b) $\delta_{ik}\, \epsilon_{ikm} = 0$ and

(c) $\epsilon_{iks}\, \epsilon_{mps} = \delta_{im}\, \delta_{kp} - \delta_{ip}\, \delta_{km} = 0$

8. (a) Obtain the generating function of Laguerre polynomials

(b) Deduce the Rodrigue's formula for $J_n(x)$.