9. (a) What are character tables? Explain the steps involved in constructing a character table.

(b) Construct the character table of the dihedral group D_3 consisting of the following three classes: E, (A,B), (C,D,F).

10. Derive the relativistic Lagrangian and Hamiltonian for a free particle.

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NOVEMBER/DECEMBER 2019

MPH21 — MATHEMATICAL PHYSICS II

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL the questions.

1. (a) Show that if $f(z) = u + iv$. is an analytic function and $\vec{F} = v\hat{i} + u\hat{j}$ is a vector, then $div\,\vec{F} = 0$ and $curl\,\vec{F} = 0$ are equivalent to Cauchy — Riemann equations.

Or

(b) Evaluate $\oint_C (z-a)^n \, dz$ where C is the circle with centre a and radius r. Discuss the case when $n = -1$.

2. (a) Obtain the solution of Laplace's equation in spherical coordinates.

Or

(b) Obtain the D' Alembert's solution for vibrating string and give its physical interpretation.

3. (a) State and prove the convolution theorem involving Fourier transforms.

Or

(b) Using Laplace transform technique, solve $y''+2y'+5y = e^{-x}\sin x$, with $y(0)=0, y'(0)=1$.

4. (a) State and prove the orthogonality theorem.

Or

(b) Prove that the set of all symmetry operations of an equilateral triangle form a group.

5. (a) Using Lorentz transformations, obtain the velocity addition formula and criticize it.

Or

(b) Derive the Doppler's relativistic formula for light waves in vacuum.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. By the method of residues

(a) show that $\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}$ and

(b) evaluate $\int_{-\infty}^{+\infty} \frac{\sin x\,dx}{x^2+4x+5}$

7. A sphere of radius a centered at O is cut into two equal halves by the xy plane. The upper part is maintained at potential $+V_o$, and the lower part is at potential $-V_o$. Find the potential at any point inside the sphere.

8. Use the method of Fourier transform to determine the displacement $y(x,t)$ of an infinite string, given that the string is initially at rest and the initial displacement is $f(x), -\infty < x < \infty$. Show that the solution can also be put in the form

$$y(x,t) = \frac{1}{2}[f(x+vt)+f(x-vt)].$$